**Double reflections over intersecting lines - Worksheet #19 – geometrycommoncore**

**NAME:** ______________________

**DOUBLE REFLECTIONS OVER INTERSECTING LINES** – Plot each of the stages of the composite transformation.

1a) \( r_{y\text{axis}} \circ r_{x\text{axis}}(\Delta ABC) \)

Circle the **resultant** transformation from \( \Delta ABC \) to \( \Delta A''B''C'' \)?

Rotation  Reflection  Translation

How did you recognize which transformation mapped A to A'', B to B'' and C to C'''?

What is the angle between the intersecting lines of reflection (the x & y axis)?  ____________

What is the \( \angle AOA'' \)?  ____________

What is the relationship between \( \angle AOA'' \) and the angle between the intersecting lines of reflection?

1b) \( r_{x\text{axis}} \circ r_{y\text{axis}}(\Delta ABC) \)

Circle the **resultant** transformation from \( \Delta ABC \) to \( \Delta A''B''C'' \)?

Rotation  Reflection  Translation

How did you recognize which transformation mapped A to A'', B to B'' and C to C'''?

What is the angle between the intersecting lines of reflection (the x & y axis)?  ____________

What is the \( \angle AOA'' \)?  ____________

What is the relationship between \( \angle AOA'' \) and the angle between the intersecting lines of reflection?

**CONJECTURE (Take a guess)**

A **double reflection** over intersecting lines seems to produce a ________________ (type of transformation).

What is the relationship between the angle between the intersecting lines of reflection and the angle that each point moves (\( \angle AOA'' \) or \( \angle BOB'' \) or \( \angle COC'' \)?)
**DOES THE ORDER MATTER?**

2a) \( r_{y=x} \circ r_{x\text{ axis}} (\triangle ABC) \)

A (7,-4)  
B (3,-2)  
C (6,-1)

Reflect over the x axis  \( r_{x\text{ axis}} (x, y) = (x, -y) \)

A’ (___, ___)  
B’ (___, ___)  
C’ (___, ___)

Reflect over the y = x line  \( r_{y=x} (x, y) = (y, x) \)

A” (___, ___)  
B” (___, ___)  
C” (___, ___)

Determine the general resultant coordinate rule for this double reflection over intersecting lines?

\( P(x, y) \rightarrow P”(_______, ______) \)

This rule should be familiar – what is it the rule for?

2b) \( r_{x\text{ axis}} \circ r_{y=x} (\Delta ABC) \)

A (7,-4)  
B (3,-2)  
C (6,-1)

Reflect over the y = x line  \( r_{y=x} (x, y) = (y, x) \)

A” (___, ___)  
B” (___, ___)  
C” (___, ___)

Reflect over the x axis  \( r_{x\text{ axis}} (x, y) = (x, -y) \)

A’ (___, ___)  
B’ (___, ___)  
C’ (___, ___)

Determine the general resultant coordinate rule be for this double reflection over intersecting lines?

\( P(x, y) \rightarrow P”(_______, ______) \)

This rule should be familiar – what is it the rule for?

How did the order impact the results of these two examples?
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The acute between these the two intersecting lines (x axis & y = x line) of reflection is 45°. In 2a, the resultant rotation was 90° and in 2b, the resultant rotation was -90°. Make a conjecture about this relationship.

In an earlier objective (G.CO.4) we discussed how rotations of a given amount have an infinite amount of co-terminal angles; for example 90° = 450° = -270° (90° + 360n). When working with the angle between the two intersecting lines of reflection it is a convention to use the acute angle value and not its obtuse supplement.

3a) \( y = x \circ r_{x\text{axis}} (\Delta ABC) \)

A (1,6) B (2,4) C (-2,2)

Reflect over the x axis \( r_{x\text{axis}} (x, y) = (x, -y) \)

A’ ( _____ , ____ ) B’ ( _____ , ____ ) C’ ( _____ , ____ )

Reflect over the y = x line \( r_{y = x} (x, y) = (y, x) \)

A’’ ( _____ , ____ ) B’’ ( _____ , ____ ) C’’ ( _____ , ____ )

Determine the general resultant coordinate rule for this double reflection over intersecting lines?

\[ P(x, y) \rightarrow P''(_____ , _____) \]

This rule should be familiar – what is it the rule for?

3b) \( r_{x\text{axis}} \circ r_{y = x} (\Delta ABC) \)

A (1,6) B (2,4) C (-2,2)

Reflect over the y = x line \( r_{y = x} (x, y) = (y, x) \)

A’’ ( _____ , ____ ) B’’ ( _____ , ____ ) C’’ ( _____ , ____ )

Reflect over the x axis \( r_{x\text{axis}} (x, y) = (x, -y) \)

A’ ( _____ , ____ ) B’ ( _____ , ____ ) C’ ( _____ , ____ )

Determine the general resultant coordinate rule be for this double reflection over intersecting lines?

\[ P(x, y) \rightarrow P''(_____ , _____) \]

This rule should be familiar – what is it the rule for?
How can you determine the size and direction (positive or negative) of the resultant rotation?

4. Determine the resultant rotation angle value from the double reflection over intersecting lines.  
(More than one answer is possible for each of these question – we will use acute angle to determine the rotation value.)

\[ r_m \circ r_n(\Delta DEF) \]

Resultant Rotation Angle Value

\[ r_m \circ r_n(\Delta DEF) \]

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Resultant Rotation Angle Value

\[ r_m \circ r_n(\Delta DEF) \]

Resultant Rotation Angle Value

\[ r_m \circ r_n(\Delta DEF) \]

Resultant Rotation Angle Value

5. Summary of relationships found in previous exercises.

a) A double reflection over two intersecting lines results in a _________________(type of transformation).

b) The angle of rotation of the double reflection over intersecting lines is exactly _______________ the angle between the intersecting lines.
c) The direction of the rotation depends on ____________________.

6. Complete the following

a) If you wanted to rotate a shape by 90° by double reflecting it over two intersecting lines, the angle between the two intersecting lines would need to be ________.

b) If you wanted to rotate a shape by 110° by double reflecting it over two intersecting lines, the angle between the two intersecting lines would need to be ________.

c) If you wanted to rotate a shape by 24° by double reflecting it over two intersecting lines, the angle between the two intersecting lines would need to be ________.

d) If you wanted to rotate a shape by 200° by double reflecting it over two intersecting lines, the angle between the two intersecting lines would need to be ________.

7. Determine the following for any point P.

a) Using the 30° angle as the angle of intersection, would \( r_k \circ r_m (P) \) be a positive or negative rotation? __________

b) Using the 83° angle as the angle of intersection, would \( r_m \circ r_j (P) \) be a positive or negative rotation? __________

c) Using the 30° angle as the angle of intersection, would \( r_m \circ r_k (P) \) be a positive or negative rotation? __________

8. Determine the following for any point P.

a) \( r_____ \circ r_____ (P) \) would result in a rotation of -60°.

b) \( r_____ \circ r_____ (P) \) would result in a rotation of 134°.

c) \( r_____ \circ r_____ (P) \) would result in a rotation of 194°.

d) \( r_____ \circ r_____ (P) \) would result in a rotation of 166°.
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9. Determine the following for any point P.
   a) \( r_k \circ r_h(P) \) would result in a rotation of \( \text{__________}^\circ \).
   b) \( r_h \circ r_j(P) \) would result in a rotation of \( \text{__________}^\circ \).
   c) \( r_h \circ r_k(P) \) would result in a rotation of \( \text{__________}^\circ \).
   d) \( r_j \circ r_g(P) \) would result in a rotation of \( \text{__________}^\circ \).
   e) \( r_g \circ r_k(P) \) would result in a rotation of \( \text{__________}^\circ \).

10. If the order that we do these reflections matters as to whether the rotation is positive or negative, why doesn’t it matter for these two composite transformations, \( r_{\text{y axis}} \circ r_{\text{x axis}}(\triangle ABC) \) and \( r_{\text{x axis}} \circ r_{\text{y axis}}(\triangle ABC) \)?

11. Complete the composite transformation so that it correct.

\[
r_____ \circ r_____ (\triangle ABC) = R_{O,42^\circ}(\triangle ABC) \quad r_____ \circ r_____ (\triangle ABC) = R_{O,-42^\circ}(\triangle ABC)
\]

12. The following question was found in the homework and two students answered it differently.

\[
r_n \circ r_m(\triangle DEF) \quad \begin{array}{c|c}
\text{Student #1} & \text{Student #2} \\
\end{array}
\]
What is the rotation angle?

Answer: -148

Answer: +212

Why are both of these answers correct?